

A TASK FROM ONLINE COMPETITION "VIVA MATHEMATICS WITH COMPUTER" FOR STEAM EDUCATION

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Abstract: *A task from the online competition "VIVA Mathematics with Computer" and opportunities to use the provided files with dynamic composition in them are presented. The results of the participants in the competition are analyzed. The emphasis is placed on using symmetries to rationally solve the problem as well as to create a similar composition with GeoGebra. The proposed content is suitable for use in learning in STEAM centers.*

Keywords: STEAM, symmetry, rotational symmetry, GeoGebra, competition, dynamic files

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1 INTRODUCTION

Games are suitable tools for the formation of the concept of symmetries among the students. Various resources in the context of STEAM are developed to support the learning of mathematics through art. Some of these resources are directly related to symmetry. As an example of the propaedeutics of geometric transformations at the

kindergarten age, finding differences in figures is suggested in (Chehlarova, 2021a). Exploring the styles of Mondrian, Escher, Andy Warhol, architecture and design, embroidery, and more in the context of mathematics and information technology, using symmetries, is developed in (Abas and Salman, 2007), (Boeva, 2021), (Chehlarova and Chehlarova, 2020), (Chehlarova, 2021), (Chehlarova and Chehlarova, 2021), (Chehlarova and Chehlarova, 2014), (Chehlarova, 2019), (Darvas, 2007), (Darvas and Farkas, 2011), (Dunham and Shier, 2021), (Jablan and Radovic, 2014), (Marchis, 2009).

The online competition VIVA Mathematics with a Computer is organized by the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences. It has been held since 2014 with students from 3rd to 12th grade (Chehlarova and Gachev, 2020), (Chehlarova, 2021b), (Kenderov et al., 2021). Each topic contains 10 tasks, the solution to which is given up to 1 hour. Its characteristic feature is the provision of auxiliary GeoGebra files (Hohenwarter et al., 2009). The goals of the competition are related to the development of students' digital competence, the promotion of files that can be considered tools for solving practical tasks, the development of spatial imagination, the development of mathematical competence, etc.

In the periodic editions of the VIVA Mathematics with Computer online competition, it is common to give the same task to different age group, but with a different set of initial data. Here we will present such a task.

2 TASK WITH A FILE WITH DYNAMIC FIGURE

Task 1 of the competition held on March 13, 2022, is about counting sections. For 3rd grade, in addition to the text "Count the segments", the condition includes a dynamic file developed with GeoGebra. A photo of the 3rd grade file is presented in Figure 1.

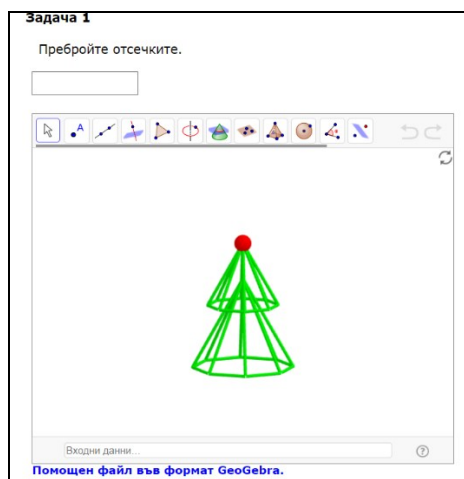


Figure 1: Task 1 for the 3rd grade of the online competition VIVA Mathematics with a computer on 13/03/2022. <http://course.cabinet.bg/index.php?contenttype=publicview&testidselectedbyuser=294>

Usually, the provided files serve to support solving the task. In this case, the file is also part of the task condition. The dynamic composition can be observed from different directions, zoom can also be used. Students are expected to notice that the composition can be considered, for example, as consisting of two parts, upper and lower, which contain an equal number of segments. Figure 2 shows two views of the composition. The first allows us to determine the n of the n -fold rotational symmetry. In the second view, the parts of the two pyramids are demonstrated.

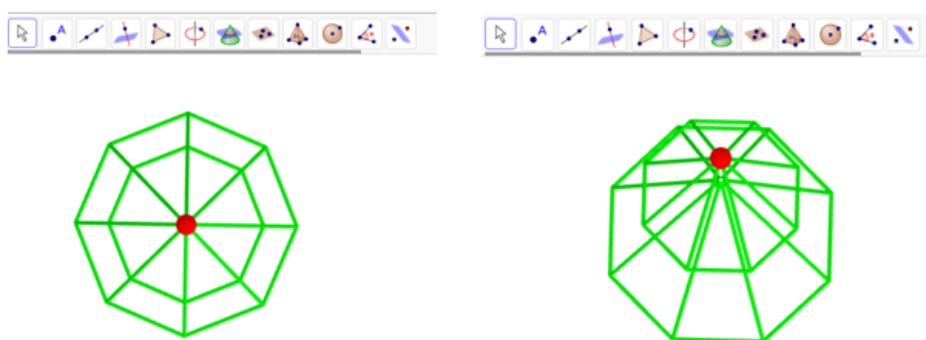


Figure 2: Two views of the composition.

One option for a rational calculation is using n of the n -fold rotational symmetry, the number of "layers", and the number of segments subject to rotational symmetry. In this case, it is enough to find the product $8 \times 2 \times 2 = 32$.

Figure 3 shows the compositions for the remaining classes.

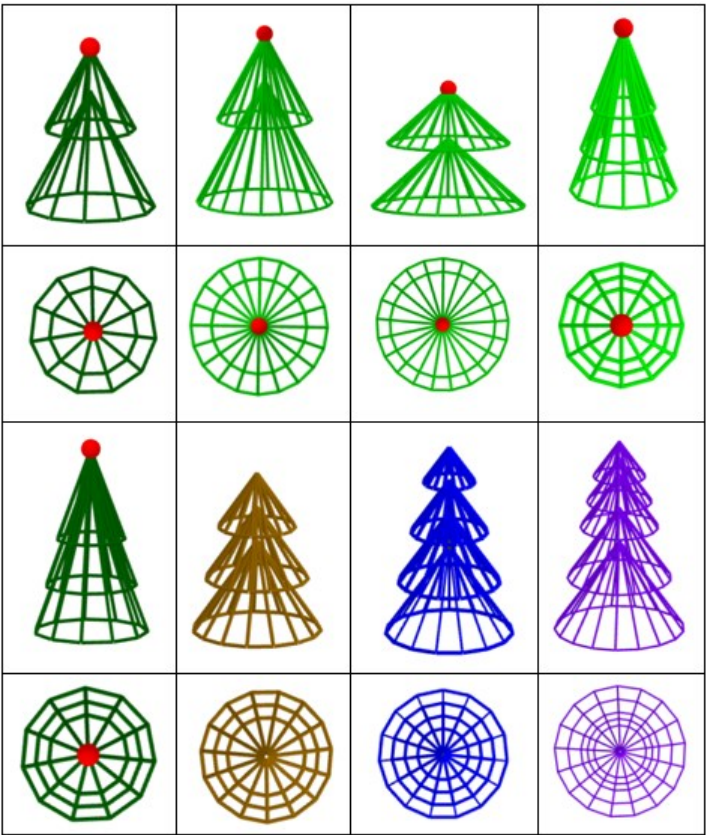


Figure 3: The compositions of task 1 in topics from 4th to 11-12th grades.

Even when using the above reasoning for a rational count, a precise count is required. Sometimes it is convenient to use the evenness of rotational symmetry both for orientation in the figure and as an intermediate check of the final answer.

The results of the participants in the edition of the competition considered here are shown in Figure 4. As usual with the results of task 1 of the previous editions of the competition, here too the percentage of those who got the correct answer is high. All

students from grades 11-12 wrote down the correct answer. The fact that some of the participants indicated half of the correct answer is also interesting (Figure 5). Some of the answers give the information that the counting was done individually for each of the segments.

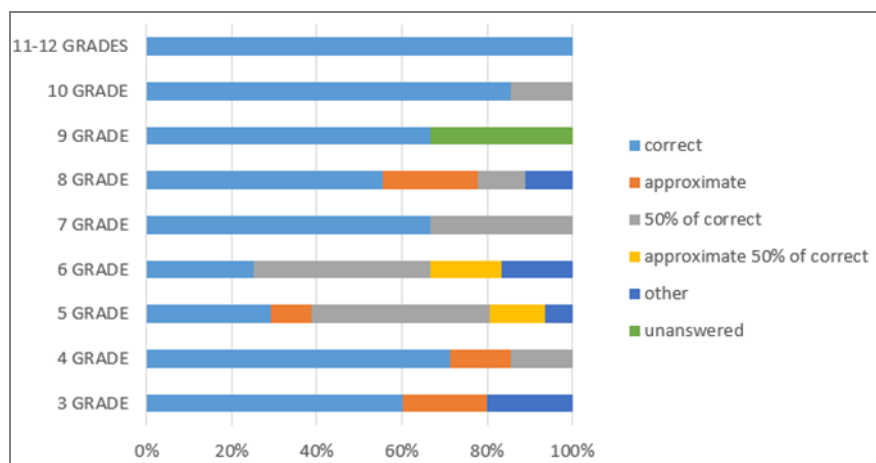


Figure 4: Participants' results for Task 1.

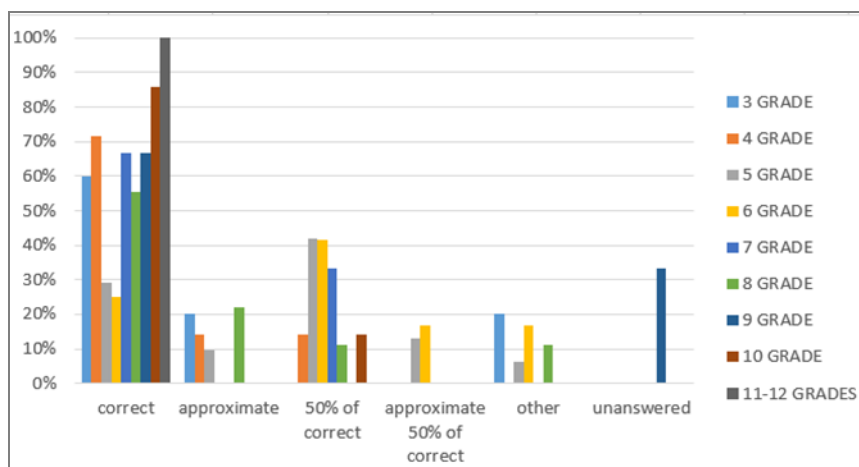


Figure 5: Participants' results for Task 1.


In this competition, when scoring free-response tasks, a target type is used, i.e. points are given not only for the correct answer, but also for one that is close to the correct answer. (Gachev, 2015).

As stated, one of the goals of the competition is the introduction of dynamic educational resources and the use of the created tools for both problem solving and software learning. In this case, the composition is associated with an image of a Christmas tree, and its use in the context of STEAM education is appropriate.


3 BUILDING A CHRISTMAS TREE IN THE STYLE OF THE FIGURE FROM THE TASK UNDER CONSIDERATION



We present several models for building the compositions in task 1. When building dynamic structures, on the one hand, the capabilities of the used software are used, and on the other, the properties of the object are used. When building dynamic constructions with GeoGebra, you can use buttons, tools, commands, knowledge of analytical geometry, and most often a combination of them.

Model 1. If we consider the composition as composed of several regular pyramids that have equal edges, i.e. of several layers, each of which is a regular pyramid, the construction can be done in this sequence (Figure 6):

- We construct two points, for example $A(0,0,0)$ and $B(0,1,0)$ - with a tool or with the commands $A=(0,0,0)$ and $B=(0,1,0)$.
- We construct a regular n -gon poly1 by using the tool  Regular Polygon or the command $\text{poly1}=\text{Polygon}(A, B, 9, \text{xOyPlane})$.

We pay attention to the fact that these two constructions can also be carried out in the plane window, and in this case, the points will have coordinates $A(0,0)$ and $B(0,1)$, and the command – $\text{poly1}=\text{Polygon}(A, B, 9)$. In this case, a regular nine-angled pyramid will be built.

- We construct the centroid J of the regular polygon with the command $J=\text{Centroid}(\text{poly1})$.
- We construct a perpendicular p through the point J to the plane of the regular polygon with the tool  Perpendicular Line or with the command $p=\text{PerpendicularLine}(J, \text{poly1})$.

- We construct a point K on the perpendicular p with the tool  Point on Object or with the command $K = \text{Point}(p)$.
- We build a correct pyramid with the tool  Pyramid or with the command $a = \text{Pyramid}(\text{poly1}, K)$.

We set transparency to 0, hiding the coordinate axes and the raised perpendicular to the base, as well as the center of the base of the pyramid.

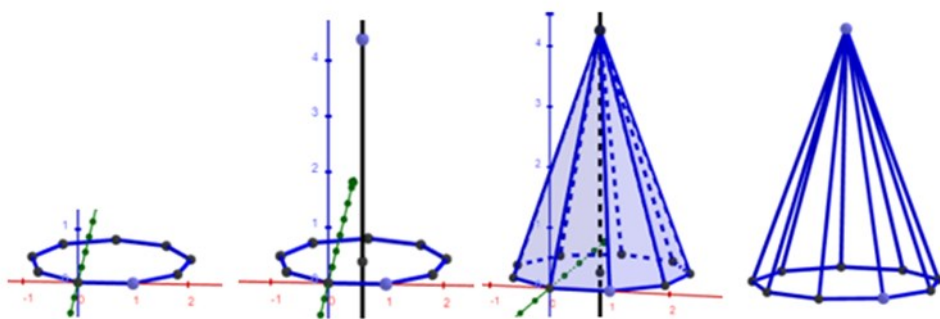


Figure 6: Construction of a pyramid according to Method 1. <https://cabinet.bg/content/en/html/d25343.html>

Step by step construction, as the construction protocol can be observed in Figure 7.

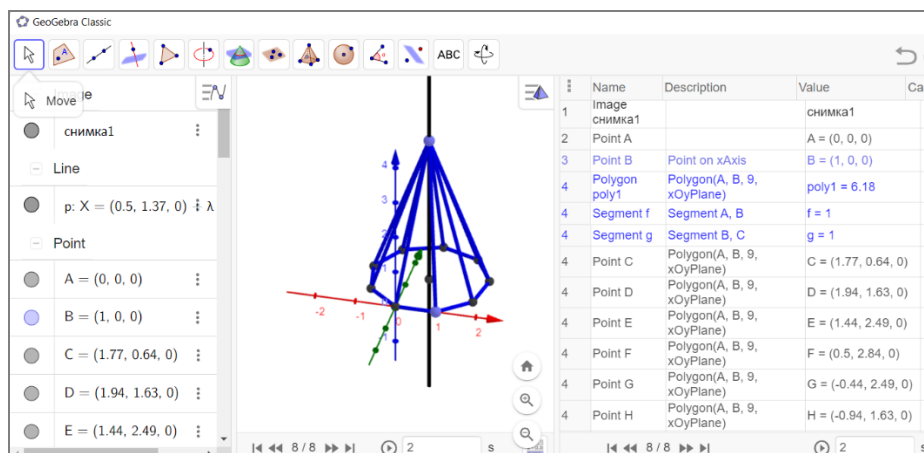


Figure 7: Model 1 pyramid construction protocol.

Analogously, the next layer can be made, using the same axis. However, here we will show the construction of the next layer with homothety (Figure 8).

- We construct a point Q from the axis p, which will be the center of homothety.
- We construct the image a1 of the pyramid a for homothety with center Q and coefficient 0.7 with the command $a1 = \text{Dilate}(a, 0.7, Q)$.

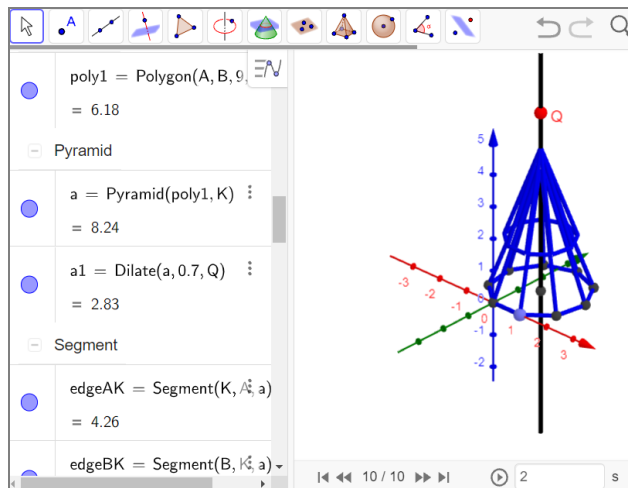


Figure 8: Constructing the second layer of the Christmas tree. <https://cabinet.bg/content/en/html/d25344.html>

Method 2. Another approach is used to build the provided files.

A slider n is created that will control the number of sides of the regular polygon. It will serve as a base of the regular pyramid (Figure 9).

- A circle is constructed with the center at the origin of the coordinate system and radius 1. We construct the origin of the coordinate system O and point $A(0, 1, 0)$ from the circle, as well as the segment $R=OA$.
- We define $b=2R \sin(\pi/n)$, which is the side length of a regular polygon.
- A cylinder is constructed with equation $(x - 1)^2 + y^2 = b^2$.

- The intersection points B and C of the cylinder with the circle are located.
- An n-gon is constructed, with adjacent vertices A and B.

After that, the construction is as described in Model 1.

This is how we present ideas for using software for building in space.

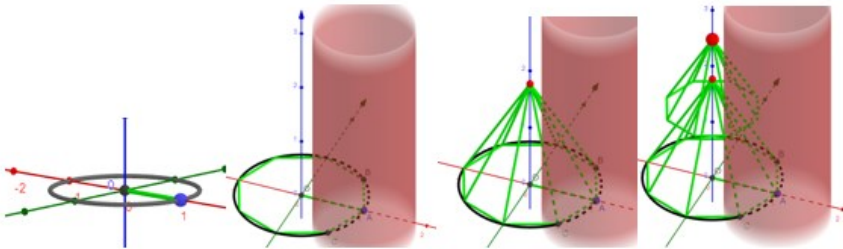


Figure 9: Construction process according to Method 2.

Method 3. For the construction as well as for solving the problem under consideration, the rotational symmetry of several sections can be used. Figure 10 shows a segment and the result of rotating one end of it around the origin of the coordinate system. For this purpose, we used a list

Sequence(Segment(A, Rotate(B, $i(360^\circ)/a$, (0, 0, 0))), i, 1, a),

where the number of turns is defined by a , i.e. the number of surrounding edges of the pyramid or the points $A(0, 0, 1)$ and $B(2, 0, 0)$ which are the ends of the line segment that is subject to rotational symmetry. In this case, one end of this segment remains stationary. Points A and B are movable, and the specific value of the parameter a can be changed, which allows changing the shape of the Christmas tree being built.

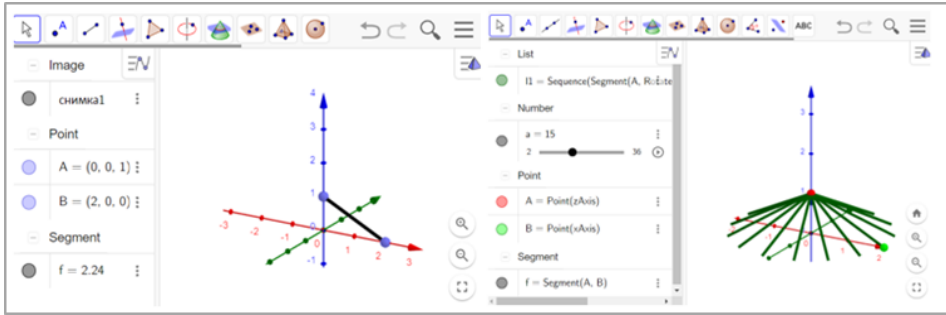


Figure 10: Construction of pyramid elements according to Method 3.

<https://cabinet.bg/content/en/html/d25345.html>

Figure 11 shows the result of applying rotational symmetry to the segment with green edges.

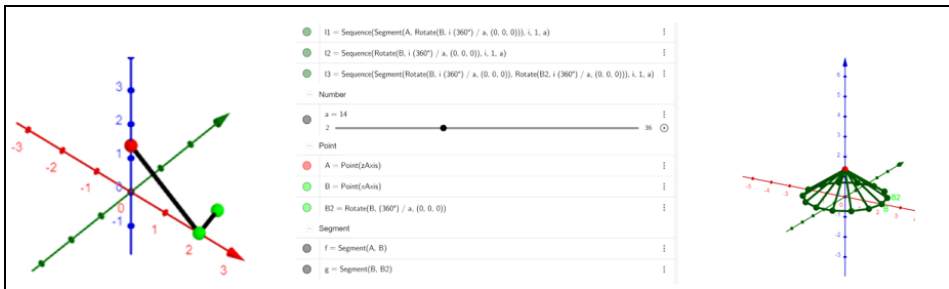


Figure 11: Constructing pyramid according to Method 3. <https://cabinet.bg/content/en/html/d25346.html>

For this purpose, point B2 and list I3 are created:

$$B2 = \text{Rotate}(B, (360^\circ) / a, (0, 0, 0))$$

$$I3 = \text{Sequence}(\text{Segment}(\text{Rotate}(B, i (360^\circ) / a, (0, 0, 0)), \text{Rotate}(B2, i (360^\circ) / a, (0, 0, 0))), i, 1, a).$$

Method 4. To build elements of the tree, it is convenient to use rotation about an axis, which is implemented in the file of Figure 12.



Figure 12: Constructing pyramid according to Method 4. <https://cabinet.bg/content/en/html/d25347.html>

For this purpose, we use a list:

$\text{Sequence}(\text{Segment}(\text{Rotate}(A, i (360^\circ) / a, z\text{Axis}), \text{Rotate}(B, i (360^\circ) / a, z\text{Axis})), i, 1, a).$

The rest of the elements of the Christmas tree can be built in the same way.

After creating the geometric structure, it follows shaping with color, animation, decoration, etc., using the capabilities of GeoGebra.

4 CONCLUSION

The topics from the past editions of the VIVA Mathematics with Computer online competition are freely accessible. Tasks are traditionally used for self-preparation for participation in subsequent editions of the competition, as well as by teachers for organizing learning activities in compulsory and optional education. When using the task considered in the article, emphasis can be placed both on the use of symmetries to find rational solutions and on STEAM education. Task files can also be used to learn the software. The ability to continue research with some of these files, including creating artistic compositions, is essential. Creating an artistic product has an additional and motivating effect on learning mathematics and information technology. The presented research is related to the study and use of the connection between science, art and education. In the context of STEAM education it is natural to seek and use the results of science and art in education.

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